# ACHIEVABLE DEGREES-OF-FREEDOM OF $(N, K)$-USER INTERFERENCE CHANNEL WITH DISTRIBUTED BEAMFORMING

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**ABSTRACT**

A distributed beamforming technique at each user pair (transmitter–receiver) is proposed in a $(n, K)$-user interference channel where $K$ user pairs are allowed to simultaneously communicate with each other among $n$ user pairs ($K \ll n$). Each transmitter sends a *single* spatial stream and each user pair minimizes generating interference to the scheduled receivers and the received interference from the scheduled transmitters via transmit beamforming and receive beamforming, respectively. We analyze scaling of $n$ to achieve $K$ degrees-of-freedom (DoF) with high probability via the proposed beamforming technique. This results show that the proposed beamforming reduces the required network size (i.e., scaling of $n$) compared to the previous results on $(n, K)$-user interference channel.

***Index Terms*—** Degrees of freedom, distributed user scheduling, transmit/receive beamforming, user scaling law, interference channel

## 1. INTRODUCTION

Recently, degrees-of-freedom (DoF) of $(n, K)$-user interference channel has been investigated in terms of the required network size for a given DoF [1–4]. In the $(n, K)$-user interference channel, it is assumed that only $K$ user pairs among $n$ user pairs are allowed to communicate with each other due to restricted resources or scheduling complexity ($K \ll n$). The authors of [1, 2] proposed *centralized* user-group scheduling (CUS) in which a genie divides the whole $n$ user pairs into $\lfloor \frac{n}{K} \rfloor$ disjoint sub-groups and then selects the sub-group yielding the maximum achievable rate. For a single-input single-output (SISO) channel, it is shown that the DoF of $d \in [0, K]$ is achievable by the CUS without power optimization if $n = \omega(\mathsf{SNR}^{d(K-1)})$[1], where $\mathsf{SNR}$ denotes the signal-to-noise ratio. For multiple-input multiple-output (MIMO) channels, it is also shown that the DoF of $d \in [0, MK]$ is achievable by the CUS without power optimization if $n = \omega(\mathsf{SNR}^{d^2+d(K-2)M})$ for $d < 2M - 1$ and $n = \omega(\mathsf{SNR}^{d(MK-1)})$ for $d \geq 2M - 1$, where $M$ is the number of antennas at each user. However, the CUS may not be feasible in distributed networks like ad-hoc networks.

For distributed networks, *distributed* user pair scheduling (DUS) was proposed for $(n, K)$-user interference channel for SISO [3] and MIMO transmissions [4]. It was shown in [3, 4] that the DUS can achieve the maximum DoF with a significantly less stringent scaling law of $n$ even with a distributed operation, compared to the CUS; the maximum DoF of $MK$ is achievable by the DUS with MIMO transmission if $n = \omega(\mathsf{SNR}^{M^2K(K-1)})$ [4]. This result was obtained when each transmitter simply sends multiple spatial streams ($M$) and each receiver adopts a zero-forcing (ZF) MIMO receiver. On the other hands, when transmit and receive beamforming is used for interference mitigation, achievable DoF and the required scaling law of $n$ corresponding to the achievable DoF have not been investigated for $(n, K)$-user interference channel.

In this paper, we propose a novel DUS with distributed beamforming (DUS-DBF) for $(n, K)$-user interference channel and investigate achievable DoF and the required scaling law of $n$ corresponding to the achievable DoF. In the proposed technique, each transmitter sends a single spatial stream with beamforming which minimizes generating interference to other user pairs and each receiver performs beamforming to minimize the received interference from the other user pairs. As a main result, it is proved that the proposed DUS-DBF achieves the DoF of $K$ if $n = \omega(\mathsf{SNR}^{(K-M)(K-M+1)})$ which is much smaller than the previous results [2, 4] for the same DoF. Our simulation result also shows that compared with the existing schemes, the DUS-DBF effectively reduces the amount of interference in the network, which validates practical merits of the proposed scheme.

## 2. SYSTEM MODEL

We consider an $(n, K)$-user interference channel embedded in a wireless dense network ($n \to \infty$) where $n$ user pairs

[1]Through out the paper, the following notations are used if two functions $f(x)$ and $g(x)$ have the following relationship: $f(x) = \omega(g(x))$ if $\lim_{x\to\infty} f(x)/g(x) = \infty$ and $f(x) = o(g(x))$ if $\lim_{x\to\infty} f(x)/g(x) = 0$.



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(i.e., transmitter-receiver pair) are randomly distributed in a two dimensional finite area and each user is equipped with $M$ antennas. Each transmitter sends an independent message for its designated receiver. It is assumed that only $K$ user pairs among $n$ user pairs communicate with each other, and each transmitter sends a single spatial stream to the corresponding receiver at each time slot. A time-invariant frequency flat fading channel is assumed and all user pairs in the network are assumed to be synchronized in time. We only consider the case that $K > M$ since each receiver can perfectly null out the interference from other transmitters if $K \leq M$.

Let $U$ be the set of indices of all user pairs in the network and $S_k$ be the index set of $k$ selected user pairs ($1 \leq k \leq K$). Since the user pair is sequentially selected for transmission in the proposed technique, $|S_k| = k$ and $S_k \subset S_l$ for all $l \geq k$. We also define $S_k^c = U \backslash S_k$ as the index set of unselected user pairs after the $k$-th step of user pair selection. After $K$ user pairs are selected by the proposed DUS-DBF, the $K$ selected transmitters send their data simultaneously, which constructs a $K$-user interference channel. Without loss of generality and for mathematical simplicity, we denote the indices of $K$ selected user pairs as $\mathcal{K} = \{1, 2, \ldots, K\}$. Then, the $t$-th received signal at the $j$-th user ($j \in \mathcal{K}$) is given as

$$\mathbf{y}_j[t] = \sqrt{\gamma_{j,j}}\, \mathbf{H}_{j,j}[t]\tilde{\mathbf{w}}_j[t]x_j[t] + \sum_{i=1, i\neq j}^{K} \sqrt{\gamma_{i,j}}\, \mathbf{H}_{i,j}[t]\tilde{\mathbf{w}}_i[t]x_i[t] + \mathbf{z}_j[t], \quad (1)$$

where $\sqrt{\gamma_{i,j}}$ ($\leq 1$) denotes the path-loss between the $i$-th transmitter and the $j$-th receiver ($i, j \in \mathcal{K}$). $\mathbf{H}_{i,j}[t] \in \mathbb{C}^{M\times M}$ indicates the channel matrix between the $i$-th transmitter and the $j$-th receiver, of which each element is modeled as an independent and identically distributed (i.i.d.) complex Gaussian random variable with zero mean and unit variance. $x_i[t]$ indicates the transmitted signal of the $i$-th user. $\tilde{\mathbf{w}}_i[t] \in \mathbb{C}^{M\times 1}$ denotes the transmit beamforming vector at the $i$-th transmitter and $\mathbf{z}_j[t] \in \mathbb{C}^{M\times 1}$ denotes an Gaussian noise vector at the $j$-th receiver where each element has zero mean and unit variance, i.e., $\mathbf{z}_j[t] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. From now, we omit the time index $t$.

## 3. DISTRIBUTED USER PAIR SCHEDULING WITH TRANSMIT AND RECEIVE BEAMFORMING

We consider both transmit beamforming and receive beamforming for each user. With transmit and receive beamforming, each transmitter sends a single spatial stream.

### 3.1. Transmit & Receive Beamforming

In DUS-DBF, each transmitter performs transmit beamforming to minimize total amounts of the generated interference from the transmitter to the user pairs already selected. Similarly, each receiver performs beamforming to minimize total amounts of the received interference from the user pairs already selected. For given transmit beamforming vectors $\tilde{\mathbf{w}}_s$ for $s \in S_{k-1}^c$, the interference generated from an arbitrary transmitter $s \in S_{k-1}^c$ to the $k-1$ selected receivers in $S_{k-1}$ is given by

$$\tilde{I}_{\mathrm{GI}}^{k,s} = \sum_{j=1}^{k-1} \gamma_{s,j} \|\mathbf{u}_j^H \mathbf{H}_{s,j} \tilde{\mathbf{w}}_s\|^2 \mathsf{SNR} = ||\mathbf{G}^{k,s}\tilde{\mathbf{w}}_s||^2 \mathsf{SNR}, \quad (2)$$

where $\mathsf{SNR}$ is the transmit power, $\mathbf{u}_j$ is the receive beamforming vector of the $j$-th selected user pair, which will be given in (10), and

$$\mathbf{G}^{k,s} = \left[\sqrt{\gamma_{s,1}}(\mathbf{u}_1^H\mathbf{H}_{s,1})^T, \ldots, \sqrt{\gamma_{s,k-1}}(\mathbf{u}_{k-1}^H\mathbf{H}_{s,k-1})^T\right]^T \quad (3)$$

where $\mathbf{a}^H$ denotes the conjugate transpose of vector $\mathbf{a}$. Using a singular value decomposition (SVD), $\mathbf{G}^{k,s}$ is expressed as

$$\mathbf{G}^{k,s} = \mathbf{Q}^{k,s}\, \mathbf{\Sigma}^{k,s} \left(\mathbf{V}^{k,s}\right)^H \in \mathbb{C}^{(k-1)\times M}, \quad (4)$$

where $\mathbf{Q}^{k,s} \in \mathbb{C}^{(k-1)\times(k-1)}$, $\mathbf{V}^{k,s} \in \mathbb{C}^{M\times M}$, and $\mathbf{\Sigma}^{k,s} = \mathrm{diag}(\sigma_1^{k,s}, \cdots, \sigma_p^{k,s}) \in \mathbb{C}^{(k-1)\times M}$, where $\sigma_1^{k,s} \geq \sigma_2^{k,s} \geq \cdots \geq \sigma_p^{k,s}$ and $p = \min(k-1, M)$. Based on (2), to minimize total amounts of interference to the users pairs already selected, the transmit beamforming vector $\mathbf{w}_s$ for transmitter $s \in S_{k-1}^c$ is determined as

$$\mathbf{w}_s = \arg\min_{\mathbf{v}} \|\mathbf{G}^{k,s}\mathbf{v}\|^2 = \mathbf{v}_M^{k,s}, \quad (5)$$

where $\mathbf{v}_M^{k,s}$ is the $M$-th column of $\mathbf{V}^{k,s}$. For given the transmit beamforming vector $\mathbf{w}_s$, the total amounts of interference generated from transmitter $s \in S_{k-1}^c$ to the user pairs selected already $S_{k-1}$ is given by

$$I_{\mathrm{GI}}^{k,s} = \sum_{j=1}^{k-1} \gamma_{s,j} |\mathbf{u}_j^H \mathbf{H}_{s,j}\mathbf{w}_s|^2 \mathsf{SNR}. \quad (6)$$

Similarly, for given receive beamforming vectors $\tilde{\mathbf{u}}_s$ of $s \in S_{k-1}^c$, the received interference at the receiver $s \in S_{k-1}^c$, that are received from the $k-1$ selected transmitters in $S_{k-1}$, can be written by

$$\tilde{I}_{\mathrm{RI}}^{k,s} = \sum_{i=1}^{k-1} \gamma_{i,s} |\tilde{\mathbf{u}}_s^H \mathbf{H}_{i,s}\mathbf{w}_i|^2 \mathsf{SNR} = ||\tilde{\mathbf{u}}_s^H \mathbf{G}^{k,s}|| \mathsf{SNR}, \quad (7)$$

where $\mathbf{w}_i$ is the transmit beamforming vector of the $i$-th selected user pair, which is given in (5), and

$$\mathbf{G}^{k,s} = [\sqrt{\gamma_{1,s}}\mathbf{H}_{1,s}\mathbf{w}_1, \ldots, \sqrt{\gamma_{k-1,s}}\mathbf{H}_{k-1,s}\mathbf{w}_{k-1}]. \quad (8)$$

Using SVD, $\mathbf{G}^{k,s}$ is decomposed as

$$\mathbf{G}^{k,s} = \mathbf{Q}^{k,s}\, \mathbf{\Sigma}^{k,s} \left(\mathbf{V}^{k,s}\right)^H \in \mathbb{C}^{M\times(k-1)}, \quad (9)$$



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where $\mathbf{Q}^{k,s} \in \mathbb{C}^{M\times M}$, $\mathbf{V}^{k,s} \in \mathbb{C}^{(k-1)\times(k-1)}$, and $\mathbf{\Sigma}^{k,s} = \mathrm{diag}(\sigma_1^{k,s}, \cdots, \sigma_z^{k,s}) \in \mathbb{C}^{M\times(k-1)}$, where $\sigma_1^{k,s} \geq \sigma_2^{k,s} \geq \cdots \geq \sigma_z^{k,s}$ and $z = \min\{M, k-1\}$. Based on (9), to minimize total amounts of received interference at receiver $s \in S_{k-1}^c$, the receive beamforming vector is determined as

$$\mathbf{u}_s = \arg\min_{\mathbf{q}} ||\mathbf{q}^H \mathbf{G}^{k,s}||^2 = \mathbf{q}_M^{k,s}, \tag{10}$$

where $\mathbf{q}_M^{k,s}$ is the $M$-th column of $\mathbf{Q}^{k,s}$. For given receive beamforming vector $\mathbf{u}_s$, the total amounts of interference at $s \in S_{k-1}^c$, received from the transmitter in $S_{k-1}$, is given by

$$I_{\mathrm{RI}}^{k,s} = \sum_{i=1}^{k-1} \gamma_{i,s} |\mathbf{u}_s^H \mathbf{H}_{i,s} \mathbf{w}_i|^2 \mathsf{SNR}. \tag{11}$$

### 3.2. Overall Procedure

We assume that time duration for pilot signal transmission is short enough, compared to that of data transmission. The overall procedure of the proposed DUS-DBF as follows:

• *Step 1 (First user pair selection with random beamforming, i.e., $k = 1$)*: Each transmitter generates a random backoff time and the transmitter having the minimum backoff time is selected and then its index is added in $S_1$. The first user pair transmit and receive data with random transmit and receive beamforming. The first user pair exchange reference signals (or pilot signals) precoded by $\mathbf{w}_1$ and $\mathbf{u}_1$, respectively, for estimating the effective (precoded) channel betweem themselves.

• *Step 2 (User pair selection with interference nulling, i.e., $2 \leq k \leq M$)*: Overhearing the exchange of the reference signals in Step 1, each user pair in $S_1^c$ determine the transmit and the receive beamforming vectors $\mathbf{w}_s$ and $\mathbf{u}_s$ according to (5) and (10), respectively. Note that when $2 \leq k \leq M$ all user pairs in $S_1^c$ can perfectly null out interference by choosing $\mathbf{w}_s$ and $\mathbf{u}_s$ such that $I_{\mathrm{GI}}^{2,s} = 0$ and $I_{\mathrm{RI}}^{2,s} = 0$. Then, each transmitter in $S_1^c$ generates random backoff time and the index of the transmitter having the mininum backoff time is added to $S_2$. The selected user pair exchange reference signals with the transmit and the receive beamforming vectors $\mathbf{w}_2$ and $\mathbf{u}_2$. Overhearing the exchanged reference signals, each user pair in $S_2^c$ determine the transmit and the receive beamforming vectors $\mathbf{w}_s$ and $\mathbf{u}_s$ according to (5) and (10), respectively. In this way, the above process is repeated until $M$-th user pair is selected, i.e., $k = M$.

• *Step 3 (User pair selection with interference minimization, i.e., $M+1 \leq k \leq K$)*: When $k \geq M+1$, user pairs cannot choose $\mathbf{w}_s$ and $\mathbf{u}_s$ such that $I_{\mathrm{GI}}^{k,s} = 0$ and $I_{\mathrm{RI}}^{k,s} = 0$. Each receiver in $S_k^c$ examines if $I_{\mathrm{RI}}^{k,s} \leq \epsilon_{k-M}$, where $\epsilon_{k-M} > 0$ is a pre-determined threshold. We call it a *receiver threshold condition*. The backoff time of the receivers satisfying the threshold condition in $S_k^c$ is set to $T_{\max} \cdot \frac{I_{\mathrm{RI}}^{k,s}}{\epsilon_{k-M}}$, where $T_{\max}$ denotes the maximum backoff time. After the generated backoff time expires, each receiver sends 1-bit signal to the corresponding transmitter, indicating whether the receiver threshold condition is satisfied or not. Once the corresponding transmitter receives the 1-bit signal from the receiver, it checks if $I_{\mathrm{GI}}^{k,s} \leq \epsilon_{k-M}$, which is called a *transmitter threshold condition*. If the transmitter threshold condition is also satisfied, then the user pair is selected and its index is added in $S_k$. The user pair exchange reference signals as in *Step 2*. If no user pair in $S_k^c$ satisfies both the receiver and the transmitter threshold conditions, *scheduling outage* is declared. Once the scheduling outage occurs, all nodes included in $S_{k-1}$ defer transmission and reset the protocol. The process is repeated until $k = K$.

• *Stage 4 (Data transmission)*: After $K$ user pairs are selected, the selected transmitters simultaneously send their data with the chosen transmit beamforming vectors, and the selected receivers decode the received data with the chosen receive beamforming vectors, which forms so called $K$-user interference channel.

## 4. DOF ACHIEVABILITY

In this section, we analyze the achievable DoF of the proposed DUS-DBF protocol in $(n, K)$-user interference channel. The received signal at the $j$-th user in (1) is rewritten as

$$r_j = \sqrt{\gamma_{j,j}} \mathbf{u}_j^H \mathbf{H}_{j,j} \mathbf{w}_j x_j + \sum_{\substack{i=1 \\ i\neq j}}^{K} \sqrt{\gamma_{i,j}} \mathbf{u}_j^H \mathbf{H}_{i,j} \mathbf{w}_i x_i + \mathbf{u}_j^H \mathbf{z}_j, \tag{12}$$

and the achievable rate of the $j$-th receiver is given by

$$R_j = \log\left(1 + \frac{\gamma_{j,j} |\mathbf{u}_j^H \mathbf{H}_{j,j} \mathbf{w}_j|^2 \, \mathsf{SNR}}{\|\mathbf{u}_j\|^2 + \sum_{i=1, i\neq j}^{K} \gamma_{i,j} |\mathbf{u}_j^H \mathbf{H}_{i,j} \mathbf{w}_i|^2 \mathsf{SNR}}\right). \tag{13}$$

**Theorem 1.** *The proposed DUS-DBF achieves the DoF of $K$ if $n = \omega(\mathsf{SNR}^{(K-M)(K-M+1)})$, where $M$ denotes the number of antennas at each user.*

*Proof.* DoF 1 is achievable if the total amount of the received interference at a single user remains *finite* as SNR goes to infinity [4]. Thus, the achievable DoF is written by

$$\mathsf{DoF} = K \cdot \lim_{\mathsf{SNR}\to\infty} \mathcal{P}_{\mathrm{DBF}}, \tag{14}$$

where $\mathcal{P}_{\mathrm{DBF}} = \mathbb{P}\left\{\sum_{\substack{i=1 \\ i\neq j}}^{K} \gamma_{i,j} |\mathbf{u}_j^H \mathbf{H}_{i,j} \mathbf{w}_i|^2 \mathsf{SNR} \leq \epsilon, \forall j \in \mathcal{K}\right\}$, and $\epsilon > 0$ which is independent of SNR for given $j$, $1 \leq j \leq K$. The probability $\mathcal{P}_{\mathrm{DBF}}$ is lower bounded by

$$\mathcal{P}_{\mathrm{DBF}} \geq \prod_{k=M+1}^{K} \mathbb{P}\left\{ I_{\mathrm{GI}}^{k,s} \leq \frac{\epsilon_{k-M}}{\mathsf{SNR}}, I_{\mathrm{RI}}^{k,s} \leq \frac{\epsilon_{k-M}}{\mathsf{SNR}} \right\}, \tag{15}$$

$$\geq \prod_{k=M+1}^{K} \mathbb{P}\left\{ \sum_{j=1}^{k-1} \gamma_{k,j} |\hat{\mathbf{u}}_j^H \mathbf{H}_{k,j} \hat{\mathbf{w}}_k|^2 \leq \frac{\epsilon_{k-M}}{\mathsf{SNR}}, \right\}$$

$$\times \mathbb{P}\left\{ \sum_{i=1}^{k-1} \gamma_{i,k} |\hat{\mathbf{u}}_k^H \mathbf{H}_{i,k} \hat{\mathbf{w}}_i|^2 \leq \frac{\epsilon_{k-M}}{\mathsf{SNR}} \right\}, \tag{16}$$



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$$\geq \prod_{k=M+1}^{K} \mathbb{P}\left\{ J_{\mathrm{GI}}^{k,s} \leq \frac{\epsilon_{k-M}}{\mathsf{SNR}}, J_{\mathrm{RI}}^{k,s} \leq \frac{\epsilon_{k-M}}{\mathsf{SNR}} \right\}, \quad (17)$$

where $\epsilon = 2\sum_{k=M+1}^{K} \epsilon_{k-M}$, $J_{\mathrm{GI}}^{k,s} = \sum_{j=1}^{k-1} |\hat{\mathbf{u}}_j^H \mathbf{H}_{k,j} \hat{\mathbf{w}}_k|^2$, and $J_{\mathrm{RI}}^{k,s} = \sum_{i=1}^{k-1} |\hat{\mathbf{u}}_k^H \mathbf{H}_{i,k} \hat{\mathbf{w}}_i|^2$. The inequality (15) is derived from the facts that $\mathbb{P}\{A+B \leq 2\epsilon\} \geq \mathbb{P}\{A \leq \epsilon\}\mathbb{P}\{B \leq \epsilon\}$ and perfect interference nulling is possible until the $M$-th user selection. The inequality (16) comes from the assumption that the transmit and receive beamforming vectors ($\hat{\mathbf{w}}_s$ and $\hat{\mathbf{u}}_s$) are designed to minimize the generated interference and received interference without a consideration of pathloss terms, i.e., $\gamma_{i,s} = \gamma_{s,j} = 1$ in (5) and (10). In (16), since the beamforming vectors are designed based on only small scale fading, the amount of interference is larger than that by the beamforming vectors considering both pathloss and small scale fading. The inequality (17) holds because pathloss is always smaller than 1. Then, using the probability that there exists at least one user pair in the network at each user selection step, which satisfies the threshold conditions, (17) can be re-written as

$$\prod_{k=M+1}^{K} 1 - \left\{ 1 - F_{J_{\mathrm{GI}}^{k,s}}\left(\frac{\epsilon_{k-M}}{\mathsf{SNR}}\right) F_{J_{\mathrm{RI}}^{k,s}}\left(\frac{\epsilon_{k-M}}{\mathsf{SNR}}\right) \right\}^{n-k+1}, \quad (18)$$

where $F_{J_{\mathrm{GI}}^{k,s}}(\cdot)$ and $F_{J_{\mathrm{RI}}^{k,s}}(\cdot)$ are the CDFs of random variables $J_{\mathrm{GI}}^{k,s}$ and $J_{\mathrm{RI}}^{k,s}$, respectively.

**Lemma 1** ( [5]). *The first order approximation of cumulative distribution function (CDF) of $J_{\mathrm{GI}}^{k,s}$ and $J_{\mathrm{RI}}^{k,s}$ for $k \geq M+1$ is given by*

$$F_{J_{\mathrm{GI}}^{k,s}}\left(\epsilon_{k-M}\mathsf{SNR}^{-1}\right) = F_{J_{\mathrm{RI}}^{k,s}}\left(\epsilon_{k-M}\mathsf{SNR}^{-1}\right) = \Psi_k^M \cdot \mathsf{SNR}^{-\rho_k^M} + o(\Phi_k^M \cdot \mathsf{SNR}^{-\rho_k^M}), \quad (19)$$

*where $\rho_k^M = k - M$, $\Psi_k^M = \beta(\epsilon_{k-M})^{\rho_k^M}$, $\Phi_k^M = (\epsilon_{k-M})^{\rho_k^M}$, and $\beta$ is the constant independent on* $\mathsf{SNR}$.

Using Lemma 1, (18) can be given by

$$\prod_{k=M+1}^{K} 1 - \left\{ 1 - \left( \frac{(\Psi_k^M)^2}{\mathsf{SNR}^{2\rho_k^M}} + 2\frac{\Psi_k^M}{\mathsf{SNR}^{\rho_k^M}} o\left(\frac{\Phi_k^M}{\mathsf{SNR}^{\rho_k^M}}\right) + o\left(\frac{(\Psi_k^M)^2}{\mathsf{SNR}^{2\rho_k^M}}\right) \right) \right\}^{n-k+1}. \quad (20)$$

Note that each probability term in (20) approaches to 0 as $\mathsf{SNR}$ increases for a *finite* $n$. However, if the network size $n$ scales at least as $\omega(\mathsf{SNR}^{2\rho_k^M})$ for each $k \in \{M+1, \cdots, K\}$, then each probability goes to 1. It is derived from the relationship that $\lim_{x\to\infty}(1 - \frac{c}{x})^x = \frac{1}{e^c}$. Therefore, if the entire network size scales $n = \omega(\mathsf{SNR}^{\sum_{k=M+1}^{K} 2\rho_k^M}) = \omega(\mathsf{SNR}^{(K-M)(K-M+1)})$, then the probability $\mathcal{P}_{\mathrm{DBF}}$ approaches to 1 and the DoF $K$ is achievable. □

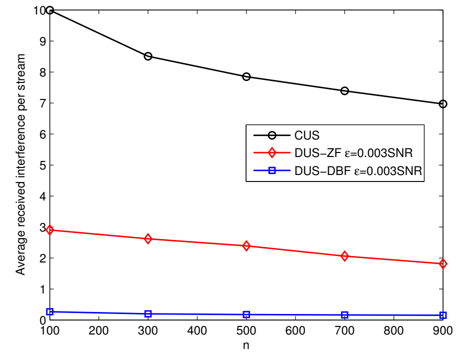


**Fig. 1**. The average received interference per stream when 6 data streams are transmitted. Two spatial streams are transmitted and three users are selected for CUS and DUS-ZF, whereas a single spatial stream is transmitted and six users are selected for DUS-DBF.

**Remark 1.** *For achieving the DoF of $K$, due to the beamforming gain, the proposed DUS-DBF requires a significantly relaxed scaling law of $n$ compared to the previous result for the single antenna case, $n = \omega(\mathsf{SNR}^{K(K-1)})$ [3].*

**Remark 2.** *For achieving the DoF of $MK$, it was shown that the DUS with zero forcing receiver (DUS-ZF) [4] requires the network size which scales as $n = \omega(\mathsf{SNR}^{M^2K(K-1)})$. On the other hand, the DUS-DBF can achieve the DoF of $MK$ by selecting $MK$ user pairs if $n = \omega(\mathsf{SNR}^{(MK-M)(MK-M+1)})$. That is, the DUS-DBF can achieve $MK$ DoF in $(n, MK)$-user interference channel. The scaling law of $n$ for the DUS-DBF is smaller than that for the DUS-ZF for achieving the same DoF of $MK$.*

## 5. NUMERICAL RESULTS

Fig. 1 shows the average received interference per spatial stream as a function of the network size $n$ when $M = 2$, $\mathsf{SNR}$=25dB, and $\gamma_{ij} = d_{ij}^{-2}$, where $d_{ij}$ is the distance between transmitter $i$ and receiver $j$. For comparison, besides the proposed DUS-DBF, the CUS and the DUS-ZF are considered, and each scheme achieves DoF of 6. This figure shows that the proposed DUS-DBF significantly outperforms the other schemes in terms of the average received interference per spatial stream.


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